Great Contribution
For World Mathematics Fundamental Education Reform
Of China's Abacus and Mental Arithmetic

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I. Analysis of Radical Obsessions in Mathematics Fundamental Education

With rapid and abrupt advancement of science and technology, knowledge economy is also booming. In worldwide education reform and especially the fundamental education reform, people pay more attention to the knowledge-based mathematics reform. Public mathematics, practical mathematics and even high-level talents training should lay stress on basic arithmetic principles and calculation skills and techniques in the fundamental education phase. But integration with information technology education even more requires a variety of algorithms. Now, the condition that children and teenagers more and more dislike the mathematic course becomes a hot and difficult point in education reforms. The analysis may make clear its major reasons:

1. Limits of numeral symbols and upright calculation mode.

The existing mathematical foundation course adopts Arabic numerals without calculation function (short for A-numerals) and the upright written arithmetic mode {short for upright mode). Both complicated problems and simple ones require each operation in mental arithmetic by means of written arithmetic and written records, causing recurrent and dull work, much brains cost and lack of enlightenment. Some countries apply calculators, with seeming relaxation. But the calculator knows little about arithmetic principles and numerals origination, leading to its failure in simple four fundamental operations. A certain upright written record of Arabic numerals and its proficiency are necessary for clear arithmetic principles and algorithms but we should abandon its disadvantages and use its advantages.

2. Obstacles of languages.

The application of Chinese language may provide 162 sentences of pithy formulas of addition and subtraction within 20 to reduce the brain burden. But other languages give little possibility for pithy formulas, leading to more burdens for children and spoiling their study interest.

3. Limit of written arithmetic.

Written arithmetic fails to accommodate the variety of algorithms, provides little space for logical thinking and imagination to cause more dullness, and fails to meet the demand of times, such as algorithm variety of information technology, abstraction of specific problems, digit compression and so on.

II. Solutions for Obsession and Creation

1. Learning from historical experience.

When Europe introduced Arabic numerals in the place of complicated and disadvantageous Roman numerals and thus emancipated both mathematics ideas and operations, the outstanding benefit was proved of selecting perfect numeral symbols for handling obsessions. Leibniz, a great mathematician and symbolist, created calculus and its symbol system, which is developed as one of the cores in modem mathematics. He applied the binary of O and I, providing conditions for creating the electronic computer, and more proving significance of symbol choices.

2. Solving obsessions genetically through Zhusuan and mental arithmetic by image of abacus.

In 1980's, chss, China fostered many children with rapidness and accuracy in abacus and arithmetic, capturing massive social attention. Some people showed opposition for it and they believed those children were forced with burdens for talents. But subsequent survey indicated none of those children suffered from poor health. Entrust- ed by the Chinese zhusuan Association and enlightened by modern research of *Symbolism,* I wrote two essays, namely *Where Secret Hidden - Preliminary Discussion on Symbol Application of Abacus and Mental Arithmetic,* and *Abacus in Perspective of Symbolism,* to elaborate that the bead -code symbol has the excusive function of automatic results through the parallel arrangement, appears easy to be accepted and applied by children with different languages and cultures, much reduces burdens of brain memory and calculation, and directly taps children's mathematic mentality, helping children spend more time on many other subjects, with which the single written arithmetic teaching cannot be paralleled.

In the time about my essays, professor Guo Qishu put forward symbolization of abacus and considered it as a key step for Abacus towards its modernization. He sorted out moving bead-code four modes, which have 26 moving bead-codes, similar to 26 Latin alphabets and a fixed algorithm. This method will cut brain cost and benefit one's life compared with memorizing 162 sentences of pithy formulas within 20. (For more details consult books such as *Zhusuan - Mathematics -Theory and Practice of Modem Abacus,* by Guo Qishu, Chen Yuguang and Liang Te- qiu.) Guo Qishu delivered his essay at the First World Conference on Abacus and pointed out, "After bringing Abacus symbols into mathematical symbolical system, the Sino-Western teaching methods will be really mixed up and superior mathematics and the mathematical teaching system will be established."

The operation in abacus and mental arithmetic in- stead of written records of Arabic numerals much benefits the mathematical foundation education reform .It can fully exert genetic advantages of Abacus symbol calculation, simplify the existing mathematical foundation education with advantages of written arithmetic, and reduce five years of integer four fundamental operations to three years education in the same quality. It has taken clear effects in experiments of 3-arithmetic teaching combination of mouth, pen and abacus and experiments of abacus and mental arithmetic entering mathematics education for thirty years, and will make great contributions including economic benefits for world mathematical foundation education reform.

IV. Function of Education Integration of zhusuan Mental Arithmetic By Image of Abacus and Information Technology

1. Ready -made picture-recording machine.

Despite complicated development of computers, teaching basic structural principles and proving its calculability require the simplest picture-recording machine. There is no essential difference on mathematical principles and arithmetic foundation between the computer and zhusuan and mental arithmetic by image of abacus; the only difference is that the computer applies electronic equipments but abacus is based on brain and hand. In teaching, the abacus white box is perceivable and easy for children's understanding.

2. Abacus and computer operation model.

The operation models of written arithmetic and computer are contradicted. Computer mathematical principles and arithmetic appear rather difficult for teaching. But abacus shares the mechanical operations and iterative application of the same storage unit with the computer. We may say abacus is the ready -made study tool to learn computer mathematical principles and arithmetic foundation. For details please consult the related works by professor Guo Qishu.

3. Incorporation of abacus.

Input, calculation, storage and output can be fulfilled simultaneously (unavailable for electronic computers), which is also the reason that abacus and mental arithmetic can cut brain - cost and remain fast.

4. Binary number.

Drive beam beads for 826 and frame beads show 173. This shows binary numbers. When the last rod of frame beads gets one more, the frame -code becomes the compliment of the beam - code. Considering the frame - code positive and the bead -code negative, addition of the positive means subtraction of the negative and vise versa. Thus addition and subtraction of positives and negatives become perceivable and natural, not only beneficial to mathematics teaching but convenient for practical use, diversifying solutions of problems. Hence, several years of exclusive teaching of positives and negatives become unnecessary, thus simplifying the key difficult problem and reducing teaching time. This is another exclusive function on calculation and education of Chinese Abacus in rational mathematics, which was explained and advocated by professor Guo Qi-shu. Written arithmetic can- not create this train of thoughts and the computer either does not bear this function for practical use, to stimulate students for active exploration of mathematics- interests. The normal experiments carried out by the Ministry of Education in Shanghai have proved: Teaching addition and subtraction of positives and negatives in grade three and four in primary schools makes students learn more easily and get enlightenment, and appears time-reducing and easy to learn, compared with written arithmetic.

5. Close combination of number and form.

Driving beads for numbers is followed with inter- changes of beads matrix, close combination of numbers and forms, and combination of abstraction and specification. This feature is much beneficial to foster students' ability of specification of abstract problems. Liu Mingfu, a researcher in Sichuan Education and Science Institute once took as an example the question " Hens and rabbits in the same cage. "The upper beads stand for heads and two lower beads close to the beam for two feet, similarly four beads for four feet. First, students are asked to drive 14 for rabbits. Then the question is presented: There are 14 heads and feet altogether for a cage of rabbits and chickens. What is the respective number of chickens and rabbits? The former 14 beads for heads remain unchanged. The former beads for 56 feet should decrease by 16. Students are asked to think first. When students think out that eight rods should reduce by two respectively. Then the answer is shown out that there are 8 chickens and 6 rabbits. Obviously, this is beneficial for abstract mentality of specific questions, to make arithmetic principles clear and make the train of thought attractive. This education function is unavailable for written arithmetic and computer. Bead matrix becomes easy point matrix when expanded outside the plate. Students are guided to study the relationship between form and number and the law of variety, which will be also beneficial for plane geometry study.

6. Diversification of algorithms of zhusuan and mental arithmetic by image of abacus.

Edition of information technology software embodies drastic competition of algorithms and disparity on effects of different algorithms for the same information. Therefore, mathematical foundation education should timely arrange practices on calculation skills for diversity of algorithms, instead of partially cutting these practices solely for fewer burdens of students. In this aspect, abacus bears more potential and appears much easier on operations than written arithmetic, deserving our more attention. For instance: abacus simplifies the complicated numeric in written arithmetic,

$$\frac{79}{732} - \frac{48}{671} = \frac{293}{8052},$$

into easy addition and subtraction of Abacus in the following chart. This reduces students' burdens, acquaints students easily with burden knowledge, improves students, interest and mental modes on diversity of algorithms, and provides computer arithmetic for direct reference.

Process Program (method)																Result													
Procedures	Left Side					Middle Side					Right Side					Left Side					Middle Side					Right Side				
I	+		7	3	2			7	3	2	+			7	9			7	3	2			7	3	2				7	9
II	-		6	7	1						-			4	8				6	1			7	3	2					1
III	+	7	3	2	0	+	7	3	2	0	+		7	9	0		7	3	8	1		8	0	5	2			8	2	1
IV	-	6	7	1	0						-		4	8	0			6	7	1		8	0	5	2			3	4	1
V	-		6	7	1						-			4	8							8	0	5	2			2	9	3

Figure : 1.

Zhu Zaiyu (1536-1611) in Ming Dynasty applied abacus to turn the nine -system into the decimal system, which was also an obvious example. For instance: Turning $(0.8376)_9$ into $(?)_{10,}$ generally, the following operation is necessary: $8 \times 9^{-1} + 3 \times 9^{-2}$ $7 \times 9^{-3} + 6 \times 9^{-4}$, but Zhu Zaiyu got the result with consecutive cross addition of numbers on the plate in Abacus, which can be expressed in modern written arithmetic: $(0.8376)_9 = \{8+(3+(7+6\times 9^{-1})^{9-1})^{9-1}\}^{9-1} = (0.936442)_{10}$. A complicated operation is turned into one with direct addition, simple and quick. This method is a

bright way for students to reduce burdens and get valuable knowledge. But it is unimaginable and unavailable for written arithmetic.

6. Abacus and digit compact technology.

Information requires digit compact technology .The structure of quinary and decimal system of abacus is a great achievement of digit compact technology. Compared with the existing teaching abacus with a string of ten beads, there is a disparity of bead moving speed and a big gap in profound level of mathematical mentality. Binary number of abacus is another form of digit compact. The abacus operation such as turning 98789 into $\overline{11}\ \overline{2}\ \overline{1}\ \overline{1}$ is also a form of digit compact. Therefore, abacus can link itself with the demand of information technology, foster students' mathematic mentality of digit compact technology, but written arithmetic and computers fail to do so.

IV. More Abundant Education Function of Abacus

1. System scale.

System is a key concept of cognition in modern times. Abacus and people form a person - bead system. This is a system model simple, easy to understand and perceivable to teach. For instance, overall functions of a system are larger than the simple sum of each part; functions of a system do not require high quality of each component; the person plays a dominant role instead of being the slave of the instrument; the plate and bead of an abacus functions the same whether made in wood or gold. This also helps teach students to regard things by essence through phenomena instead of superficial complication or simplicity, nobleness or cheapness, and to foster scientific spirits of being true to facts and being brave to exploration and creation.

2. Chinese abacus can acquaint weak -intelligent children, the illiterate and the blind with arithmetic.

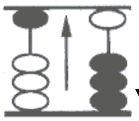
Because an abacus expresses numbers and digits through beads and rods, and shows the result automatically through parallel arrangement beads, specific and tangible, so that the weak - intelligent, the illiterate and the blind can recognize numbers through beads and rods and fulfill four fundamental operations. This also reduces a series of brain burdens in abstract numerals study and makes study easy and interesting.

3. Balanced development of the left -brain and the right brain.

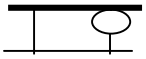
In written arithmetic, there is no arithmetic function of a pen, and brain arithmetic is needed, causing complicated programs, heavy memory burdens, and more application of the left-brain for abstract mentality. Zhusuan and mental arithmetic by image of abacus apply parallel arrangement bead -code for results, and share the same operation model with the computer, which belongs to image thinking with more application of the right brain. The left -brain and the right brain can get balanced development in Sino -Western mathematics integration.

4. Logical mentality of Abacus

Some people believe that the prevalent European western mathematics concludes the whole of mathematics; it has a perfect strict axiom deduction form, which is beneficial to sort out condition and make deduction; it pays no attention to Chinese mathematical original system, not to say seemingly simple Abacus. But we might as well further discuss logicality of abacus. zhusuan sets its places by rod, with one bead up and four down on each rod separated by frame; its strict logical system makes abacus users unconsciously form strict logical ideas, more cater- mg to the demand of modern information technology .European mathematics applies syllogism but zhusuan forms the four- step method including dialectical logic .For example: 4 + 7 deduction . Western Arithmetic



Big Prerequisite: The result of addition counts from the added.
Small Prerequisite: 4 + 7
Conclusion: Count 7 from 4, and then get 11.
Chinese Arithmetic
General Prerequisite: Addition requires moving beads towards bead.
Positive Prerequisite: 4 + 7
Negative Prerequisite: The amount of frame beads is smaller than7.
Conclusion: The frame beads are more than 7 and beads moving towards the beam are fewer than 7. Drive reverse 3 beads as the complement of 7, and then automatically get 11.



Obviously, strict Sino-Western integration is common and practical. Another example:
Deduction in Western Arithmetic
Big Prerequisite: If a part is cut off, the rest is fewer than the former.
Conclusion: Three angles are left. (Unreal)
Deduction in Chinese Arithmetic
General Prerequisite: If a part is cut off, the rest is fewer than the former.
Positive Prerequisite: A rectangle is cut off with an angle.
Negative Prerequisite: The number of angles is different with acreage.
Conclusion: There are five angles. (More than the former amount)
Obviously, application of abacus and study of its logical principles will make students form stricter logical mentality.

V. Sino -Western Integration and Exerting Functions of Abacus and Abacus and Mental Arithmetic: Best Choice for Mathematical Foundation Education Reform

In short, if the prejudice is cast aside and Sino - Western integration is conducted, the arithmetic function of bead -code symbols can genetically discard complication, brain burdens and dullness of written arithmetic. Bead -code symbols can also express integers, decimals, positives, negatives and even some figures flexibly so as to enlighten students' mathematical thinking. When the computer is unavailable, Abacus can be used to teach mathematical principles

and arithmetic foundation of the computer and strengthen students' interest on exploration of mathematics and information technology. This is in line with national condition for developing countries to catch up the developed. Away from this, the idea of cutting study time, reducing students' burdens and learning knowledge of modern meanings and value for good quality will go against expectation. This is not imaginary assumption but the experience of the countries such including the US and the UK.

Wu Wenjun, academician of the Chinese Academy of Science, said in his book *Collections of* Wu Wenjun, "To realize mathematics modernization, we must not only learn from the advantages western -based axiomatic methods, but also cherish Chinese ancient heritage, learning from historical mechanical methods." "The pith of our ancient mathematics is the mechanical ideal and method, quite complying with the demand and condition of modernization. "Weyl, a western modern reputable mathematician said in his essay *My Idea to Axiomatic Method and Structural Method,* "The majority of modern mathematics research is based on the subtle integration of the structural method and the axiomatic method. We should pay attention to the interwoven relationship between them".

Fully taking advantage of Chinese Abacus, abacus and mental arithmetic complies with objective demand of the worldwide mathematical origin and development trend. People should be called on to "pay attention to their inter-woven relationship. “Massive experiments on zhusuan mental arithmetic by image of abacus have proved abacus symbols genetically ease the difficulty of written arithmetic, reduce burdens and increase efficiency for its integration with other functions, and appear economical. Hence, I show my opinion for more discussion on it.

Courtesy:

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