



Brief Introduction to Development History Of Abacus and Mental Arithmetic in China

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I will make a brief introduction to the history of abacus and mental arithmetic in China, classification of periods and characteristics in each period, especially the history of mental arithmetic. People never discussed them before. Now I just make a preliminary exploration, welcoming any criticism and correction if some mistakes emerge.

China's abacus, with a history of three thousand years since the Western Zhou Dynasty (about 1000 BC) has witnessed a long way of algorithm evolution. Mental arithmetic depends on the then prevailing algorithm in each period and hence enjoys a long history. But ancient books have not kept detailed records, so we can just base our analysis on a few history records.

I. Counter-Calculation Period (Ancient Bead-Calculation) (Before 8 Century AD)

We call a beam – and - rod abacus a modem abacus, and those before it ancient bead - abacuses such as unearthed beads in Xizhou Dynasty and “Abacus” in *Arithmetic Chronicle*.

Bead-calculation (Abacus) and counter-calculation existed simultaneously for a long time. The former originates from stone calculation and the latter from stick and thin branches calculation, both stemming from original algorithms. According to some records in ancient books, this period seems to be concentrated on counter - calculation while bead -calculation didn't have its original method and had to share the same algorithm with counter -calculation. For example, its addition and subtraction are the same as the prevalent counter -calculation and share the similar method with the modem bead-calculation except for lack of a pithy formula. As for multiplication and division rules, clues can be found in the last paragraph of *Arithmetic Chronicle*.

Zhen Luan, an ancient mathematician, mentioned multiplication and division of stitch - calculation. He said, "In accordance with the rule of multiplication, each number is rearranged in a position. The upper *one* multi- plies with the lower one and put the result in the middle. Put the dividend on the bottom and the result on the top. It is also the rule *of* calculation in three parts". It is totally unanimously with that in counter-calculation. As one of the fourteen algorithms, bead -calculation applies the same method with stitch -calculation. So, three bead-calculating plates lined together from top to toe can offer multiplication and division. This method is inconvenient for bead -calculation so it has blocked development of bead -calculation.

In a word, algorithms in ancient bead -calculation are the same as those in counter-calculation. So we name this period "counter -calculation system period".

When human beings had calculating ability, mental arithmetic also came out. In ancient times, if people had no stone or stick for simple calculation, they calculated by image of stones and sticks; this is mental arithmetic. During this period, records in ancient books suggest images of sticks enter people's mind and lead to mental arithmetic, so we call it mental arithmetic by image of sticks. Lao-tse said, "People of good calculation use no calculating sticks. " That is to say, people who are good at calculation needn't use calculating tools such as calculating sticks. This also suggests counter-calculation can be proficient enough to be free from calculating tools, and be fulfilled in mind, in other words by mental arithmetic by image of calculating sticks. *Arithmetic Chronicle* considers better calculation " as " mental calculation free from algorithms. "Zhen Luan explained, "To be free from algorithms means not to use calculating instruments, instead, to calculate in mind. " This is also mental arithmetic by image of calculating sticks. In this period, there are two representatives of mental arithmetic by image of counters, Ji Ran and Sang Hong-yang. The two persons were considered as representatives when people talked about those of best calculating ability. So there is a saying, "Yan and Sang, masters of mental arithmetic. "Yan here refers to Ji Ran, a native in Pushang Caiqu in Spring and Autumn Period, with a surname Xin, given name Yan, style Wen-zi, and an alias Ji Ran for his proficiency on calculation. When he served as an official in Yue Kingdom, he put forward seven proposals such as valuing circulation and pursuing evenness, hence helping Yue Kingdom beef up to defeat Wu Kingdom and finally become one of the five powers .He was the teacher of Fan Li who applied his tragedy to do business and became a man of means. Sang refers to Sang Hong -yang .He was born in a businessman's family in Luoyang and was appreciated by Han Wu Emperor and promoted for the emperor's company in palace for his proficiency on mental arithmetic. Because he had special ability of mental arithmetic, Han Wu Emperor often demanded him to calculate national financial and economic accounts. At last he put forward some economic policies to strengthen the kingdom such as state monopoly of salt and iron and made the Western Han Dynasty a superpower in Asia. During this period, many calculating tools emerged and so did mental arithmetic by image of other calculating tools, such as Nine-Palace and One Calculation by Zhao Da, a native in Wu kingdom in Three Kingdoms Period.

Characteristics of Arithmetic in this period:

1. Many calculating tools co-existed but applied the common not original algorithms .In about six hundred years from Han Dynasty to South and North Dynasty, people found the speed of counter -calculation was too slow to meet the demand of economic development; hence calculating tools experienced much transformation so that thirteen calculating tools emerged according to records of *Arithmetic Chronicle*. On the other hand, algorithms witnessed no reform and kept using multi -position method. Therefore, transformation of calculating tools didn't work and counter-calculation still got the priority.
2. Mathematical equation were expressed by rectangular matrix and read from top to bottom, with a direct) bearing on China's ancient writing mode.
3. All algorithms tried to be transfolllled into addition and subtraction. For example, consecutive division was changed into consecutive subtraction and mutual multiplication and elimination of equation was turned into direct division. The reason is that both bead -calculation and counter -calculation are good at addition and subtraction.

4. Counter- calculation entered an advanced calculating phase but bead-calculation still remained in the phase of four fundamental operations.

II. Transition Period (Preliminary Period of Abacus) (about the 8th – 13th century AD)

Transition period saw transition of counter- calculation to bead -calculation, from the mid Tang Dynasty to late Sung Dynasty, with a history of about five hundred years.

After 755 AD in mid Tang Dynasty, manor system merged. Manors of some independence might undertake various economic forms including handicraft industry and agriculture so market exchanges increased and business developed forward drastically. In order to meet this kind of social demand, calculating techniques had to be improved. Therefore, bead-calculation reform was promoted and ancient bead-calculation began to shift to modern bead-calculation (abacus) during this period. Because of birth of abacus, a trend of researching modern bead - calculation became uprising in mid Tang Dynasty. Multiplication evolved from addition outside to "pursuing one" and division evolved from "subtraction outside "to" pursuing one" and to late adding percentage and return -division. This both met the demand of counter –calculation and created a good condition to establish new bead -algorithm.

In this period, the mental arithmetic remained to be mental arithmetic by image of calculating sticks. For example, Wei Pu, born blind in Huai'an Jiangsu, was a mental arithmetician in Sung Dynasty. He taught himself mathematics and had special ability especially the ability of mental arithmetic. He could even calculate the time of solar and lunar eclipses. He was recommended by Shen Kuo to edit Fengyuan Calendar .He orally did with multiplication and division with no mistake. He could recite the vertical or level number table in almanac rapidly so that people couldn't follow the numbers by their bright eyes. He was blind so most of the calculation was fulfilled in mental arithmetic.

Characteristics of Arithmetic in this period:

- I. A lot of swift rules of multiplication and division came out.
 2. Calculation in three parts evolved to single arrangement of beads. Direct product and quotient by real numbers emerged.
 3. Many calculating pithy formulas came out, so did the peak period of pithy formulas, which had a direct bearing on flourishing culture of prose in Tang and Sung Dynasties.
 4. Multiplication experienced transition from front multiplication to back multiplication, and division developed from estimating quotient by mental arithmetic to setting quotient by pithy formulas.
1. Abacus still could only deal with four fundamental operations.

III. Regularization Period (Classical abacus and mental arithmetic Method) (About the 14th Century AD to 1949 AD)

This period ranged from Yuan Dynasty to founding of the People's Republic of China, with a history of above six hundred years. After calculation reform in the last period, the bead abacus gradually came into shape and the algorithm became regularized. Its regularization was marked by the appearance of *Comprehensive Calculation* by Cheng Da-wei, final accomplishment of addition and subtraction pithy formula, when multiplication concentrates on head multiplication and division on return division. Upon five hundred years, it saw little changes.

At that time, mental arithmetic came into being, which can be proved by records about Tang Shun -zhi in Tang Dynasty, and Shi Yue -chun in Qing Dynasty, who were both good at calculation. Tang Shun -zhi (1507- 560 AD) with a style Ying -de, alias Jing -chuan was native in Wujin. He won the first in the examination in length year of Emperor Jiajing's Reign and took a high position in Qing Dynasty. He was good at calculation. He made achievements on struggle against pirates but died at 54 because of illness from onerous work. Once he went to Luzhou and met calculation about corn and cost. He got a dozen of people who could use abacus, and asked them to give numbers respectively. He moved beads up and down, just getting their numbers with process of calculation unseen. Within less than two hours he finished all the accounts of the government. At that time, the veterans of bead -calculation were shocked by his accuracy and promptness. Actually, what he applied was abacus and mental arithmetic. Shi Yue -chun, styled Qing -fu, was a native in Jiading Jiangsu. In 1860, he, together with Ding Qu -zhong in Hu Lin -yi's government in Wuchang, saw Ding apply simultaneous equation system in *Arithmetic Chronicle* to solve "the problem of a hundred chickens." Then he began to edit two volumes of a book on solving the one -hundred -chicken problem in 1861 to spread this method. When doing so, he was blind but he was still able to use an abacus to calculate in abacus and mental arithmetic. With the help of his son, who recorded calculating process, he finally finished his book.

Characteristics during this period:

1. All four fundamental operations applied pithy formulas. Addition applied the upper pithy formula, subtraction withdrawing pithy formula, multiplication nine -nine pithy formula and division nine -return pithy formula, up-one pithy formula, collision -return pithy formula and nine -nine pithy formulas.
2. The rule of fixed position of multiplication and division was completed. Direct multiplication and division from multiplicand and dividend came out.
3. Abacus algorithms developed towards advanced operations, such as solving linear equation, extraction of the square root, higher degree equation.

4. Abacus algorithms remained comparatively stable and nearly unchanged with introduction of west written arithmetic and bamboo chip arithmetic. Some people put forward ideas to integrate abacus and bamboo chips arithmetic or integrate abacus with written arithmetic.

5. A lot of books on abacus came out. The most popular one was *Comprehensive Calculation* (1592). The one of the highest level was *Principle of Algorithm* (1524).

I. Reform Period of Algorithms (1950-2000)

After founding of the People's Republic of China, people were in more urgent-need of culture and science and technology. In urban area, with gradual development of industry and commerce, and increasing demand of financial calculation, abacus learning became rather important. In rural area, with development of co-operative system and demand of accountants in production groups, abacus was also indispensable. Therefore, it came out a trend of abacus learning of massive people. In order to grasp abacus, quickly many abacus experts put forward some plans of abacus reform. For example, Yu Ziyi, Zhong Luzhai, Yu Jieshi, Chen Zibei and Hua Yinchun and so on put forward their opinions to reform old abacus, stated to eliminate addition -subtraction method with pithy -formula, return -division and to apply assembling and complement addition-subtraction and the way of division by multiplication table, and later put forward multiplication of empty abacus from left figure and division for many quotients and so on.

During this period, mental arithmetic was listed in the courses in primary schools and 3 - arithmetic mathematics teaching was put forward. In 1960's, Shanghai opened a pilot region in Chongming County and then expanded to Xinhe County. At that time, it was put forward to "fully make use of abacus as a calculating tool with mental arithmetic as basis and written arithmetic as emphasis, to add efforts to 3 -arithmetic combination." "In 1972, abacus expanded to Shangcheng Hangzhou for pilot teaching experiment. It was put forward to "reform written arithmetic, promote mental arithmetic on the basis of abacus, and to fulfill organic combination of three kinds of arithmetic." The former idea was mainly for abacus learning but the latter was beneficial for development of abacus and mental arithmetic.

Assessment and Selection of Ancient and Modern Abacus and Mental Arithmetic published by the Committee of Abacus Methods Principles of the Chinese abacus Association at the end of the last century summarized all kinds of abacus calculation and selected the four fundamental operations, evolution and other algorithms available to abacus and mental arithmetic.

Characteristics during this period:

1. Pithy formulas were eliminated (mainly addition -subtraction pithy formula and return -division pithy formula).

2. New algorithms were put forward, such as addition -subtraction of symbolic arrangement, addition -sub- traction of glancing many rows, multiplication of empty abacus from left figure, speaking out quickly a unit's product.

3. Complements and negatives were applied.

4. Summarizing books on Abacus came out. The famous ones were *General Dictionary of World Abacus and Assessment and Selection of Ancient and Modern Abacus*.

V. Period of Abacus and Mental Arithmetic (2000-)

From 1980's, abacus and mental arithmetic has caught people's attention. Despite its gradual expansion, people show different ideas to abacus. In this century, Abacus workers have realized that the direction for Abacus is abacus and mental arithmetic. In the preface of *General Dictionary of World Abacus* written in 1994, I said, "It created a new way for simple abacus to develop abacus/ mental arithmetic. This may mark a new phase of China's abacus development. " In the last century, abacus remained the core; but in the new century by image of abacus mental arithmetic is the key. Abacus is only a model for development of abacus and mental arithmetic. Therefore, this period can be called the period of by image of abacus mental arithmetic.

It is an inevitable result for abacus historical development to enter the period of abacus and mental arithmetic.

Abacus has shown two functions in the history .One is to calculate and the other is to develop intelligence. Because its calculating speed is faster than ancient counter -calculation, abacus has taken place of counter -calculation. Meanwhile, it keeps the collective method of number expression in counter -calculation, hence remaining and even promoting its advantages beneficial to mental arithmetic. Goujian, king of Yue Kingdom, held Ji Ran in regard and Wu Emperor in Han Dynasty regarded Sang Hong -yang .The important reason was that they were found to have a high level of mental arithmetic be- hind which lay extraordinary wisdom. Surely, they be- came the leading officials to manage their country .In the past, abacus functioned as a calculating instrument in the history, but in the present electronic era, this function of abacus plays in a minor position. If its function of calculation is preserved without further development, abacus will be weeded out by history as counter -calculation. Then, what is the way out? The way out is to develop its function of tapping intelligence. By doing so, we can train out new talents of high intelligence.

Abacus, as a model of abacus and mental arithmetic, itself has some exclusive advantages unavailable to many other calculating instruments.

1. Variable numerals system: Under the system, fundamental operations start from high degree, so it is fundamental operations start from high degree, so it is convenient for mental arithmetic.

2. Quinary and decimal system: The amount of calculating components decreases to a half of the abacus with a string of ten beads. It is easy for memorizing numbers and convenient for brain reflection.

3. Rectangular matrix position system: Whether counter -calculation or abacus, both apply rectangular matrix (m lines and n rows in counter -calculation, n rows in abacus) on number arrangement. The property of each element in the rectangular matrix in operations depends on its fixed position. Hence, the method that word symbols define properties is

disannulled. Operations are simplified into variety of rectangular matrix. This will be beneficial for mental arithmetic.

4. Pithy -formula program system: Pithy formulas guide operations and programs. Nine - nine pithy formula is a feature of Chinese calculation and much easier than foreign multiplication. The pithy formula of speaking out quickly a unit's product appears more simple and faster on multiplication. All of these pithy formulas are beneficial for children and are a powerful tool of abacus and mental arithmetic.

Compared with other calculation instruments, abacus is prone to develop into mental arithmetic:

1. Compared with written arithmetic: Written arithmetic features changes of abstract symbols and is difficult to be reflected in people's mind. Whereas abacus features changes of practical symbols and it is easy for people to conduct image thinking. The former is a fixed code, leading to manifold calculation, onerous memory task and assiduous brain burden. Abacus is a variable code. Results can be got in a change of original code without any code reset. The less storage of memory task is convenient for mental arithmetic.

2. Compared with the other 12 calculation instruments that have been eliminated: The most fundamental advantage of abacus is high speed of changes and number recognition. Because of its own method that the number 5 is expressed with all the 5 beads down, counter - calculation is slower than abacus on number recognition and operations. The other calculating instruments are not collective and bear no numerical value themselves, which is shown by their positions and colors. Due to difficult number recognition, it has a much lower speed, which naturally brings about a lower speed of mental arithmetic. Some people once applied Taiyi Calculation instruments in, teaching practices. Despite its some good effects, it will be certainly inferior to abacus if it develops into mental arithmetic. The same is true with the so-called point - bead machine, a disguised Liaozhi calculation (one of the 14 ancient calculation methods, calculating instruments needed.)

3. Compared with the computer: The computer operations proceed inside, with changes of electronic units. It cannot be modeled by human's brain so computer calculation cannot develop into mental arithmetic.

Therefore, in accordance with the development history of calculation instruments, abacus is the right one for mental arithmetic.

In line with the development history of abacus algorithms, reforms of algorithms in the last century paved the way for by image of abacus mental arithmetic; and especially, selection of ancient and modern algorithms created good conditions for it. Certainly, algorithms that are more suitable for by image of abacus mental arithmetic will be created in the practice of by image of abacus mental arithmetic, which can be named algorithms of by image of abacus mental arithmetic by image of abacus. This is also a new task for our experts on algorithms research.

The 3 -arithmetic combination in the last period has promoted the development of by image of abacus mental arithmetic by image of abacus. But at present, written arithmetic is still learned in primary schools. How to promote combination of by image of abacus written arithmetic is a new task in our educational research. I maintain that abacus and written arithmetic

should not be separated from each other and written arithmetic should not go back to its old way just because we develop mental arithmetic by image of abacus.

In short, mental arithmetic by image of abacus is the result of historical development. There are many research tasks in front of us, which are the new tasks for our mental arithmetic by image of abacus workers.

I believe abacus and mental arithmetic has a very bright prospect.

Courtesy:

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